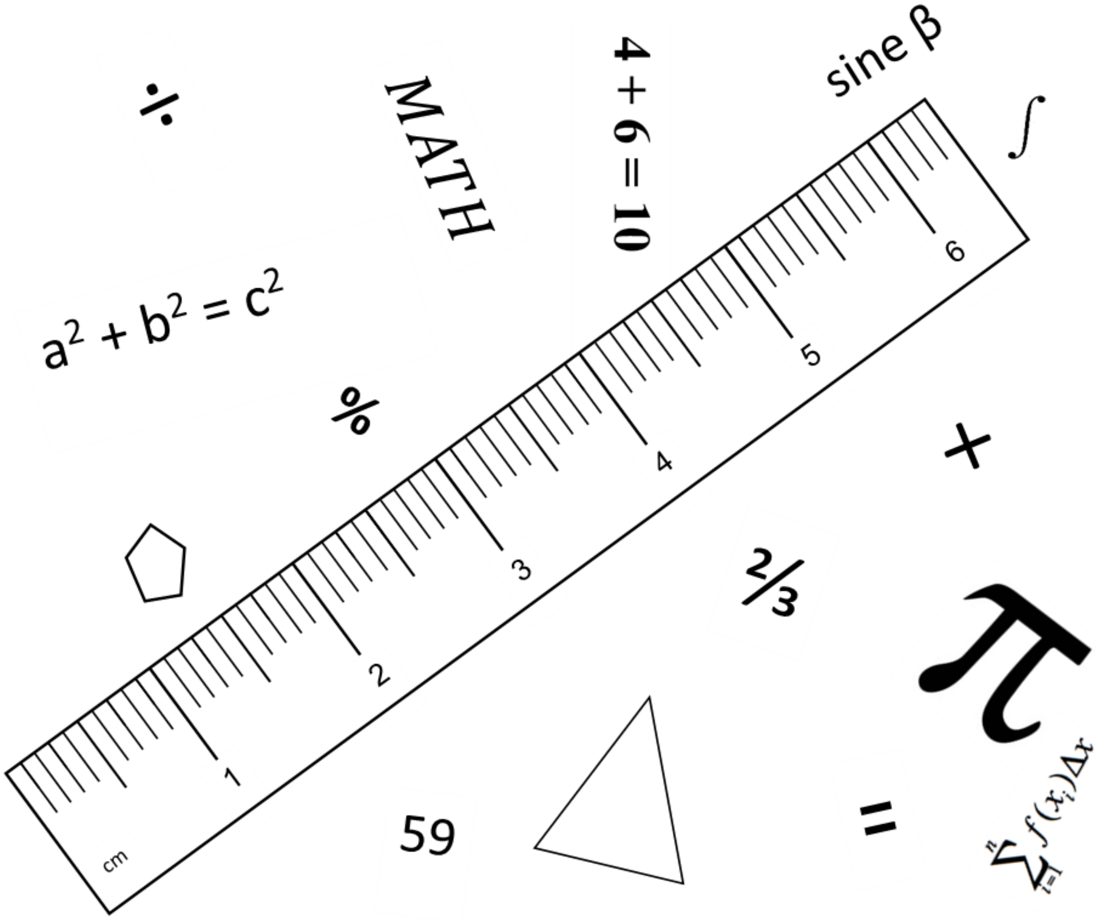
Poor Numeracy Skills

and

The Walls Come Tumbling Down



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Executive Summary

This white paper's focus centers on the paramount importance of mastering math numeracy skills in the elementary school years and their impact on subsequent grade levels. When arithmetic math fact (e.g. single digit addition, subtraction, multiplication and division facts) and math processing skills (e.g. place value, rounding, etc.) are not sufficiently developed, the deficiency directly affects middle and high school math courses and is a significant factor in limiting student performance and success.

A series of grade level problems and their solutions are presented and analyzed to illustrate the dependent nature of numeracy skills. These problems include the following grade levels: third (3^{rd}), seventh (7^{th}), high school algebra and a first-semester university calculus course. At the conclusion of each math example, there is a listing of the specific numeracy skills that must be mastered if students are to successfully engage in correctly solving that grade level problem. If students have not mastered and are not proficient at one or more of those prerequisite math numeracy skills, there exists a high probability that students will not consistently and correctly solve grade level problems of this nature. Numeracy skill gaps disproportionately and negatively affect economically disadvantaged students eventually in the form of diminished problem solving prowess. The lack of grade level mathematics proficiency of our most at-risk students becomes public when school and district academic data is released. After the data is analyzed, the comparison of the inequitable education achievement between children of poverty and their more affluent peers and the infamous 'achievement gap' is born. However, if the numeracy (and literacy) skill gaps are rectified, the achievement gap disappears as well.

This analysis is not new, and it is not a revelation to any educator that possesses a strong fundamental basis in mathematics and instruction. However, the reason that these skill gaps are not addressed when they are a primary factor in academic student outcomes is both frustrating and confusing to the author. After more than two decades of work in the public schools, it has been the author's professional experience that far too many elementary classroom teachers and administrators do not comprehend the gravity of both math processing and math fact skill mastery. A lack of educator understanding in ensuring that students possess prior and current grade level mastery of math numeracy skills impairs student learning in two primary areas. First, students struggle with proficiency of higher order math skills that are dependent upon a scaffolding of prerequisite numeracy skills. Second, students lacking prior grade and current grade level numeracy skills experience difficulty solving grade level application (word) problems. Students' working memory is taxed as numeracy skill deficiencies consume inordinate amounts of their processing power. Students must engage tremendous effort and energy when computing low-level computations that were never satisfactorily mastered and often fail to recognize grade level number sense patterns and connections. In their innumerate situation, problems appear to be 'magically' solved as numbers appear in the solution without a clear sense of origin or need. Consequently, students cannot view the problem as a whole with discrete parts that integrate into a global solution; instead, the discrete parts – individual math numeracy skill gaps – overwhelm their problem solving efforts.

As expected, the situation is exacerbated as elementary students' transition to both middle school and high school and grade level math skills and applications continue to be layered upon prior grade level skills. As newly introduced skills and applications are cumulatively stacked upon dependent prior grade level math skill deficiencies, the end result is not only student and teacher frustration, but all too often, resignation and/or defeat.

However, with the advent of the computer and computer programs as a student tracking and distribution vehicle, this situation need not continue. A computer can individually track large numbers of students and a daily numeracy system is easily implemented to ensure student mastery of both grade

level and prior grade level math processing as well as math fact skills. With a differentiated daily numeracy program and spaced repetition (instructional) system, the age-old math dilemma of poorly founded numeracy math skills can be permanently solved. The power of a daily numeracy program and a spaced repetition system is not only the instructional efficiency, but the programs are also effective for any math standard (e.g. Common Core (CC) or Texas Essential Knowledge of Skills, TEKS). An additional feature of these two systems is their viable independence from any adopted or selected math core curriculum; hence, a school or district can select any core program that fits their academic or philosophical needs. Simply put, both the daily numeracy program and spaced repetition (instruction) system are designed as platform independent methodologies.

A highly efficient and effective daily numeracy program that dramatically heightens student mathematics performance is Formative Loop. This mathematics numeracy program is a daily, differentiated program that affords first grade through eighth students progressive numeracy development – addressing both grade level and prior grade level math processing skills as well as math fact proficiency. Formative Loop is a blended paper and pencil assessment and a digital tracking and monitoring system. It can be implemented as either a classroom, grade level or a schoolwide daily numeracy program.

Spaced repetition system (SRS) instruction is a dynamic and effective addition to enhance classroom teaching. There is nothing to purchase to implement a SRS; it is an instructional methodology. A white paper extolling the specific aspects of this instructional system may be downloaded for free at the website listed in the footer. When an SRS methodology is implemented in conjunction with Formative Loop, student outcomes in mathematics are significantly improved.

Poor Numeracy Skills

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It was in my late twenties when I realized the importance of skill mastery – not just in math, but in general. My university degrees in Architectural Engineering, Accounting and Finance were nothing more than a foundation of math, writing, science and reading skills that were applied to specific fields of college study. The rudimentary academic skills learned in elementary school and built upon in middle and high school were pressed into broader applications during my university undergraduate years at the University of Texas. Those cumulative and dependent skill sets provided economic avenues for the next three decades of professional work in structural engineering, governmental accounting and public education.

From a formative educational perspective, I did not realize or fully comprehend the dramatic impact that quality public schooling had on my life until I was professionally employed. During a conversation with a colleague on a hydraulic issue that was impacting my structural engineering project, an off-handed comment was uttered about the hydraulic radius of the piping system. His end of the project was the hydraulics and mine was the structural area; however, it suddenly occurred to me that the vast majority of the skills we were discussing to solve a fairly difficult engineering problem had been learned by the time we were both in ninth or tenth grade. I became acutely aware that if a person did not possess mastery of those basic math and science skills, the probability of successfully solving this hydraulic/structural engineering problem would be so close to zero that it would be zero. In short, the final engineering solution required university level knowledge, but high school math and science skill set mastery.

The above engineering scenario underscores the paramount importance of general skill mastery in all core subjects during a students' public school years. Skilled educators realize that skills in all subjects are dependent upon prior grade levels, and that their year of educating students will soon be yet another prior year of learning as student progress to higher grade levels. Hence, those educators press and demand mastery of their subject content. Public school teachers who require mastery of fundamental skills with grade level applications afford their students career choices later in life. Those students may or may not select a lucrative technology field livelihood, but at a minimum, their students are given career options due to a sound skill foundation in both math and science.

This short paper is intended to illustrate specific points concerning rudimentary mathematics. First, students must be held accountable to possess mastery or immediate proficiency of both math process skills (i.e. place value, even/odd numbers, multiples, rounding, etc.) and the four math fact operations (i.e. single digit addition, subtraction, multiplication and division). If students do not possess grade level skill proficiency, it not only negatively impacts students in current grade level skill mastery and problem solving prowess, but those skills continue to adversely impact dependent math skill learning in subsequent grades. For example, students who are not proficient at fundamental arithmetic skills not only struggle with dependent arithmetic skills and current grade level problem solving, but as the student progresses into algebra in seventh and eighth grades, those arithmetic skills significantly impair algebraic skill and problem solving mastery as well.

Second, a presentation and analysis of grade level problems ranging from 3rd grade, 7th grade, high school algebra and a first semester university differential and integral calculus problem illustrate the rudimentary importance of dependent math skill mastery. Each grade level problem will be presented, solved and all prerequisite math skills will be listed that were essential in arriving at a solution. Importantly, if any one or two of these dependent math skills were not previously mastered, students are highly unlikely to solve a grade level problem. As expected from the initial analogy in the second paragraph above, each grade level's math problems are no different to the professional civil engineering hydraulic and structural engineering situation presented. The scenarios are only different in the age of persons but not the required skill dependencies to achieve outcome success.

3rd Grade Math Problem

The word problem in Figure 1 is typical of a third grade level math problem. The problem appears on the surface a straightforward math problem that is easily solved, and if a child possesses the required numeracy skills, it is a relatively easy problem for a typical third grader to solve. There are a couple salient points that should be discussed. First, the student must read the problem and realize that the problem has more numbers than are needed (e.g. extraneous data) and that the problem is an estimation problem by the key word 'about.' Second, after selecting only the needed data, the student must have background information in place value and possess mental 'fixity' of whole numbers relative to one another on a number line. Many Title 1 third grade students struggle with whole number sense with regard to the comparative magnitude of whole numbers. Simply put, they do not readily and mentally grasp the 'fixity' of an integer's physical location on a whole number line. Third, the student must correctly round the numbers to the nearest 10 <u>before</u> the two numbers are subtracted, and the student must know that rounding convention of the number 5 in the one's digit – rounds up to the nearest 10, not down. Lastly, the student must correctly subtract the numbers – knowing basic math facts and computation skills – double digit numbers subtracted, in this case. Additionally, students should realize that subtracting two numbers is a 'difference' between two numbers – in effect, the total number of spaces between the subtrahend and minuend. It is a process to learn, holding the numeracy skills in their working memory and then, applying the skills all at one time in an application. Finally, the student must also be taught to systematically solve a problem like this as well as evaluate the reasonableness of the solution. All of these skills may appear easy to most adults, but these skills sequentially applied is both a developmental and often a challenging process for many beginning intermediate students.

> Jack, Sam and Jill collected sea shells on the beach early Saturday morning. Although the temperature was 91 degrees on the sandy beach, Jack found 37 shells, and Sam collected 24 shells. Their sister, Jill, picked-up 65 shells. When they returned home, their mother asked them, "About how many more sea shells did Jill find than Jack?"

> <u>Solution</u>
>
> $$\begin{array}{rcr} 65 & \Longrightarrow & 70 \\ -\ \underline{37} & \Longrightarrow & \underline{-\ 40} \\ & & 30 \end{array}$$
>
> Jill found about 30 more sea shells than Jack.

Figure 1

<u>Required Numeracy Skills:</u> whole number place value, whole number lines, whole number magnitudes, rounding, subtraction facts, computational skills and evaluating the reasonableness of the solution.

7th Grade Math Problem

The seventh grade problem shown in Figure 2 clearly illustrates the progression of area math problems compared to their conceptual and pragmatic introduction in third grade. The seventh grade area problem requires students to compute the area in two different regions of the garden (shown in Figure 3) and then sum the two separate calculations to compute a total area. The number of dependent math skills from prior grades that are required to solve this problem is also deceptive. Students must understand the area concept as well as the pragmatic calculation for area using both double digit multiplication and decimal multiplication of a rectangular polygon. However, those are final computations in the solution process. The solution in this problem requires dimension translation of the opposite side of the parallelogram to compute the missing value of the width of the smaller rectangle. The missing width is calculated using basic addition and subtraction facts, whole number and decimal place value, and subtraction concepts involving decimals with regrouping. The student should also have an understanding of approximate distances with metric magnitudes (e.g. the length of a meter and half a meter). For a student to evaluate the reasonableness of his or her solution, the student must be numerically competent in rounding the dimensions and estimating the areas. For example, the student could estimate the area of the larger of the two rectangles by multiplying 15 m x 15 m – a relatively known math fact of 225. Then, the student could sum 225 and the product of 16 and 2 for a reasonable estimation of the available planting area.

Students could also have solved this problem by computing the total area of the parallelogram and subtracting off the area of the 'Rock Monument.' This solution technique requires many of the same skills except translation of the unknown dimension of the smaller rectangle. Regardless of the solution technique the number of prerequisite numeracy skills is significant, and the dependent math skills become apparent in comparison to an elementary level area problem. Finally, this problem is much more difficult if larger numbers were used including decimal multiplication of area to the hundredths place value. The 'Rock Monument' could be comprised of an "L" shaped polygon in lieu of a rectangle. Those changes would require students to possess and apply more numeracy skills and computational work in their solution. In other words, the area problem shown in Figure 2 is one of the "simpler" area problems, yet it still requires a significant number of prerequisite numeracy skills

Andreas' garden is 21 meters by 16 meters. He wants to plant flowers and vegetables. He cannot plant in the Rock Monument section of the garden, as shown on the diagram. What is the area of land that is available to plant in his garden?

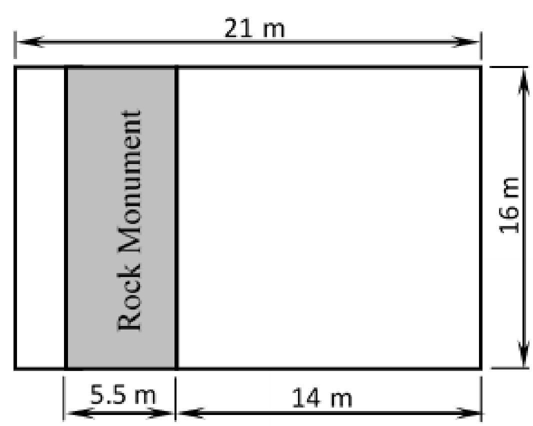

Figure 2

Solution:

Multiply 14 m x 16 m. 224 m^2
Add 5.5 m and 14 m. 19.5 m
Subtract 21 m – 19.5 m. 1.5 m
Multiply 1.5 m x 16 m. 24 m^2
Add Areas. 224 m^2 + 24 m^2
Total Area equals <u>248 m^2</u>

Figure 3

<u>Required Numeracy Skills:</u> whole number and decimal place value, rounding whole numbers and decimals, (addition, subtraction and multiplication) basic facts, linear translation of dimensions, subtracting and regrouping of decimals, multiplication of multi-digit whole numbers and decimals, addition of multi-digit numbers, area concepts and area computation of parallelograms, understanding of metric magnitudes (e.g. meters) and evaluating the reasonableness of the solution.

High School Algebra Math Problem

Algebra is a basic and progressive extension of arithmetic mathematics. The vast majority of time, algebraic math success is wholly dependent upon students' elementary school mathematics prowess. Introductory algebraic concepts usually begin in middle school years in 7th grade. Rational numbers, inequalities, proportions and solving equations linear equations for one variable are typical topics covered in the first courses of algebraic development.

As middle school students progress into high school algebra, algebraic instruction transitions into more complicated function analysis. For example, students learn to identify and solve general relationships in both linear and quadratic equational forms: $Ax + By + C = 0$ and $Ax^2 + Bx + C = 0$. Students are also taught to correctly graph their equations (and solutions) in a two dimensional Cartesian coordinate system to demonstrate understanding of the physical meaning of the equational relationships

In Figure 4 below, two linear equations are stated and labeled as Eq. 1 and Eq. 2. In high school algebra, a student is expected to be able to simultaneously solve these two equations and compute the x and y coordinates – the coordinate intersection of the two lines. An algebraic solution is also shown in Figure 4 providing a step-by-step sequence of this process. It should be noted that there are other methods to solve these two linear equations for a singular coordinate solution. One such method uses linear algebra and specifically, Gaussian Elimination, that is highly efficient. However, invariably, a high school student initially learns to solve two linear equations by isolating one of the variables and then employs back substitution to compute the other variable. The latter solution technique was selected and is depicted step-by-step in Figure 4.

There are myriad numeracy skills that are required to correctly solve the two linear equations in Figure 4. One of the primary arithmetic skills is a thorough understanding of the equal (=) sign – first introduced in the primary grades in elementary school. Students are adding, subtracting or dividing by single digits to each side of an equation to ensure that the relationship is equivalent throughout the solution process. For example, $2x = 12$, both $2x$ and 12 are both divided by 2 so the equational relationship remains equivalent. The reasonableness of the solution is evaluated using the Cartesian plot in Figure 5. Students must be able to competently plot both linear equations on the Cartesian plot by varying the values of ‘x’ and determining an associated ‘y’ value. However, since these equations are both linear and only two points are required to define a line, it is possible to zero each “x” in Eq. 1 and 2 and the location where point ‘y’ crosses the vertical axis is immediately known. The same process can be used to compute where each line crosses the horizontal ‘x’ axis. After plotting both equations 1 and 2 (i.e. ‘x + y = 10’and ‘x - y = 2’) in Figure 5, the computational coordinate solution of (x, y) equal to (6, 4) – the intersection point of the two lines is visually shown to be correct.

Another math skill that is used repetitively in the solution process is the mathematical substitution of numbers/expression in a variable. The fact that any equivalency may be inserted into an equation when the value is equal is a revelation to many students. However, it is invaluable that students understand the substitution process in simple linear equations as in this example, so when equations become more complex, students readily accept this process as a permissible tool in the algebraic process.

The elusive obvious arithmetic numeracy skills are ever present when solving two linear equations as shown in Figure 4 and 5. Students are using their knowledge of single digit math facts, positive/negative numbers, place value and plotting of x and y coordinates on a Cartesian graph. The student must be able to perform these operations with absolute confidence and with minimal mental – numeracy effort. Again, if these skills are not founded to mastery proficiency levels, the student is stopped in the solution process and their working memory is mired in the minute computations and not focused on the global mathematical conceptual and physical process.

x + y = 10 *Eq. 1*

x - y = 2 *Eq. 2*

<u>Solution:</u>

x - 2 = y (Isolating ‘y’ in Eq. 2

x + (x – 2) = 10 Substituting Eq. 2 in Eq. 1

2x – 2 = 10

2x = 12

x = 6 x-coordinate

6 - y = 2 Substituting ‘x = 6’ into Eq. 2

6 - 2 = y Simplifying

4 = y y-coordinate

(x,y) coordinate = (6,4)

Figure 4

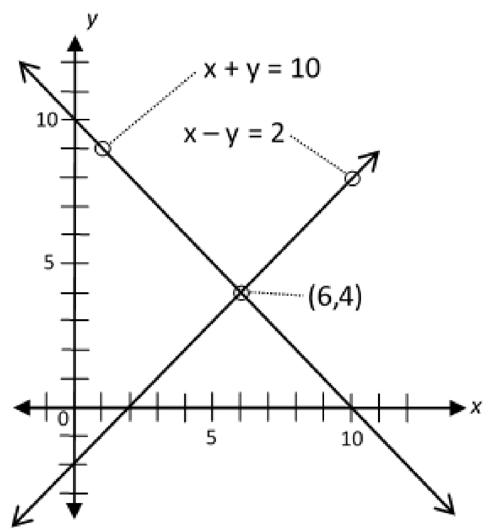

Figure 5

<u>Required Numeracy Skills:</u> whole number place value, equal sign concept and meaning, (addition, subtraction and division) math facts, understanding of positive and negative whole numbers, linear equation concepts and applications, substitution equivalencies of whole numbers and expressions, two dimensional Cartesian Coordinate System concepts, plotting (x, y) rectangular coordinates, and evaluating the reasonableness of the solution – physically plotting both linear equations to confirm figure 4 coordinate solution.

First Semester University Calculus Problem

The last type of math example that will be presented is either an advanced high school mathematics course or a typical first semester college differential and integral calculus problem. The concept of integration and computing area between bounded parameters or intervals is not a new or complicated idea. This process was first accredited to Isaac Newton in the 17th century. Although the definite integral as shown in Figure 6 appears at first glance as difficult, in reality it is not. The solution process utilizes a methodology described in the second part of the Fundamental Theorem of Calculus.

In the specific situation illustrated in Figure 6, the Theorem is computing the area bounded by the two linear equations $y = 3x + 2$ and $y = 0$ over the inclusive interval $0 \leq x \leq 4$. After integration of the function $3x + 2$ (note that the horizontal function $y = 0$ was subtracted), the interval parameters of $x = 4$ and $x = 0$ are substituted to compute the total area of 32.

$$\int_0^4 3x + 2\, dx \Rightarrow \frac{3x^2}{2} + 2x\Big|_0^4 \Rightarrow \left[\frac{3(4)^2}{2} + 2(4) - \frac{3(0)^2}{2} + 2(0)\right] \Rightarrow \left[32 - 0\right] \Rightarrow 32$$

Figure 6

The physical meaning of the definite integral is depicted in Figure 7 and assists students in a physical understanding of the basic calculus. The area is computed for four units (i.e. $x = 0$ to $x = 4$) along the x-axis bounded by $y = 3x + 2$ and $y = 0$. The region of this area makes a trapezoid that is parsed in figure 7 into a small rectangle and a right triangle, respectively. The area of each of each polygon is computed – 8 for the small rectangle and 24 for the right triangle for a total sum of 32. Graphing the solution – shown in Figure 7 – is as essential at this level of mathematics as when students use tactile manipulatives when learning addition and subtraction concepts in the elementary primary grades to ensure student understanding of the actual mathematics mechanics. Simply put, the graph in figure 7 is the analogous pictorial manipulative when a student is learning differential and integral calculus. Otherwise, students integrate functions and compute numerical quantities, but they do not possess a physical understanding of his or her solution.

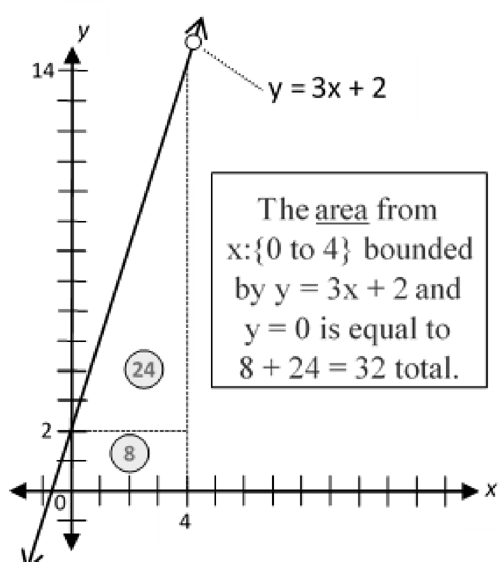

Figure 7

As in the last couple examples, there are a myriad of numeracy math skills in this process. In the computation process, student understanding is required of the conceptual and pragmatic use of the Theorem of Calculus, substitution of boundary conditions in the equation, and the use of exponents and basic mathematics operations. The graphing solution in Figure 7 requires many of the same math skills as were needed in Figure 6 of the high school algebra problem. The student should also be able to compute the areas of both a rectangle and a triangle. Finally, the summing of the two areas (i.e. 8 plus 24) yields the total area and serves to check the computations in figure 6 – integrating the linear functions y = 3x + 2 and y = 0 between x: {0, 4}.

<u>Required Numeracy Skills:</u> whole number place value, (addition, subtraction, multiplication and division) math facts, positive/negative whole numbers, Fundamental Theorem of Calculus theory and pragmatic implementation, addition and multiplication of multi-digit whole numbers, linear equation concepts and applications, substitution equivalencies, Cartesian Coordinate System concepts, plotting (x, y) rectangular coordinates and finally, area concepts and area computation of parallelograms and triangles.

Conclusion and Final Comments

As expected, the list of numeracy skills to correctly solve higher grade level math problems increases in length with each new grade level math application. More importantly, the basic arithmetic numeracy skill mastery and proficiency students learn in elementary school are paramount to ensure complete understanding of dependent higher level math skills or applications. When students have not mastered the basic arithmetic skills in both math processing and math facts, those students often become mired down in those same basic skills and are unable to globally work problems with multiple step solutions. For example, in reference to the third grade problem above, the author has worked with multiple children who do not possess the ability to correctly round numbers to the correct value due to a lack of place value and/or whole number line skills. The math word problem immediately regresses to remedial lessons on whole number place value and number lines in lieu of solving the original estimation problem.

As should be expected, innumerate students are not able to correctly parse grade level application word problems into discrete skills, and if the skill deficiencies are not remedied, they languish and retard mathematics progress in subsequent grade levels. For example, as middle and high school students are confronted with grade level mathematics – but possess prior grade level dependent skill deficiencies, mathematics often appears *magical* to them during the direct teach of a math lesson. Numbers 'appear' on the board 'out of thin air' with no logical connection or rationale since they do not possess mastery of prior grade level numeracy skills to identify their origins.

During my university years as a civil engineering student at the University of Texas, I tutored freshman and sophomore undergraduates in differential and integral calculus. The conceptual elements of calculus are not difficult to grasp; in fact, the majority of students readily comprehended an overarching perspective of that level of mathematics. Where these students invariably succumbed to the rigor of their university mathematics coursework was in deficient skill mastery and understanding in arithmetic, geometry, algebra and trigonometry from their public school years. Their tutoring session for their calculus course would transition into an algebraic, arithmetic or geometric refresher course – analogous to the third grade student's situation with place value and whole number lines presented above. It is important to note that it was more than apparent that these freshman and sophomore students had little recall of mathematics principles from their seventh or eighth grade middle school math courses. However, the more amazing aspect of this situation is that these students were readily admitted to the University of Texas from their high schools. It is disheartening to consider the large group of students who were **not** admitted to the University and understood basic mathematics even less than these students.

This paper began with two licensed professional engineers collaborating to solve a hydraulics and a structural engineering problem on an urban bridge project. The discovery that a vast majority of the math and science skills that were employed to successfully navigate this problem were mostly learned by the time both engineers were in the ninth or tenth grade was surprising to me. However, after my professional experiences teaching and administratively directing instruction over the last two decades in the public schools, it is not surprising in the least. In my professional opinion, the most apparent pedagogical mistake in arithmetic mathematics is a lack of thorough understanding of the essential need for math skill process and fact mastery. An elementary, middle and high school student is capable to solve any problem successfully based on their skill level proficiency. If a student possesses high skill mastery, the student is highly likely to acquire the application process of presented mathematics. Conversely, if the student does not possess only one or two of the required numeracy skills to solve a given problem, his or her ability to solve it correctly and readily understand the solution is significantly impacted.

Prior to advent of computers, classroom teachers had no efficient and effective way of knowing and tracking whether students had retained previously taught math skills. They were also in a position to speculate which prior grade level math skills students were proficient and which ones were not. However, there are differentiated daily numeracy programs for both elementary and middle school that are currently available that remedy this classroom situation. An effective program on the market today is Formative Loop. It is an inexpensive and ubiquitously daily program that covers nearly every grade level numeracy skill from grades one through eight as well includes prior grade level math skills in each grade level sequencing. The Formative Loop numeracy program is a standalone product, but when it is used in combination with a spaced repetition (instructional) system, student learning outcomes are dramatically heightened. These two methodologies are effective for either math standard (Common Core State Standards or Texas Essential Knowledge and Skills) and are independent of the selected core curriculum. Additional information can be obtained from Formative Loop, and the spaced repetition system (SRS) instructional process is available for immediate and free download at the website address in the footer of this document. This process tool and numeracy resource are both efficient and effective for any socioeconomic status school – it only takes the effort and implementation to ensure student mathematics success and equity for all students.