Large Whole Number Multiplication and Division Diagrams

Extracting a diagram that provides a physical model of the story or word problem

3^{rd} – 8^{th} Grade

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Executive Summary

Regardless of the state standard strands for multiplication and division applications, many students have difficulty conceptualizing the physical model of the multiplication or division computations with large whole numbers. In short, many students do not generally transition their mathematical understanding from a basic math fact to an analogous computation using larger numbers. For example, students learn that 4 x 9 = 36, and that arithmetic fact physically represents 4 equal groups of 9 **<u>or</u>** 9 equal groups of 4 for a basic multiplication group model. Fundamental multiplication can be also represent an array or area grid model, and that a 4 x 9 or a 9 x 4 array or grid contains 36 total elements.

Again, many students do not conceptually transfer physical representation from basic fact models (e.g. group, array or area) to larger whole numbers that are typical 4^{th} grade, 5^{th} grade and middle school multiplication equations (i.e. 24 x 54 = ?). Thus, a significant number of students do not recognize that group, array or area models signifying either 4 x 9 or 24 x 54 possess the *<u>EXACT the same physical representation</u>*.

Additionally, intermediate and middle school teachers frequently encounter students' inability to identify the arithmetic operation when solving a story or word problem. Simply, students are unclear if the story problem is a division or multiplication solution. This situation can occur despite students possessing computational and algorithmic proficiency with large whole numbers (e.g. 2 x 3 multiplication or long division using 2-digit divisors).

A pedagogical tool (diagram) exists that solves both issues for students, so that they are able to both conceptualize and understand the mechanics of either a multiplication or a division equation regardless of the magnitudes of the whole numbers. In fact, this simple diagram ties the basic math fact group, array and area models to larger whole number computations. Since multiplication and division are the same equation with differing missing values, the diagram for each of the two operations is the same. Consequently, after creating the diagram, students are able to recognize the missing quantity in the equation leading to the correct operation as well as conceptualizing the process. The methodology in this white paper describes a pedagogy that affords students to conceptualize the operation type (i.e. multiplication or division) and a conceptual understanding of that process despite working with large whole numbers.

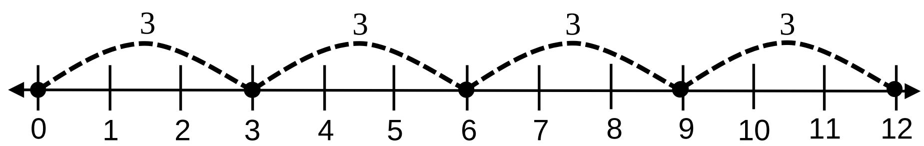
Finally, it is imperative to note that students must possess rapid recall of all four-math fact operations. This task is not as demanding as an entry-level educator may believe. Using a structured approach described in white papers that can be downloaded (free) at the website listed in the footer, students will master their basic facts. Once that is accomplished, learning two by three multiplication or long division is only dependent upon adequate practice.

Large Whole Number Multiplication and Division Diagrams

Student memorization of basic math facts of all four operations is essential arithmetic knowledge. The complete mastery and understanding of the math fact physical models should also be a pedagogical priority from initial tactile lessons to paper-pencil understanding. Students should learn multiplication as a repetitive adding **<u>group model</u>** using number lines and/or basic diagrams. Multiplication can also be represented as an **<u>array</u>** or **<u>area</u>** physical representation; hence, those models must be given specific instruction. Each of these basic multiplication models is shown below for both clarity and reference.

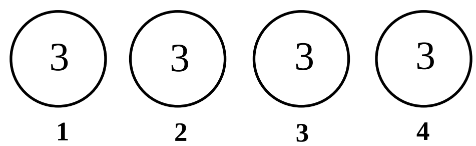
<u>Group Models:</u>

A.



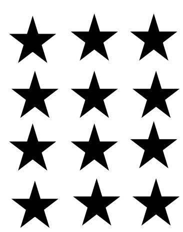
4 equal groups of 3 equals 12. Or, 4 x 3 =12

B.



4 equal groups with 3 in each group equals 12. Or, 4 x 3 =12

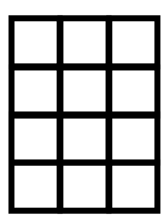
<u>Array Model:</u>



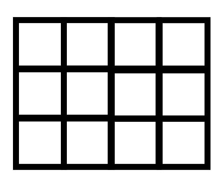
4 equal rows of 3.
Or, 4 x 3 =12

3 equal rows of 4.
Or, 3 x 4 =12

<u>Area Model:</u>



4 by 3 grid.
Or, 4 x 3 =12



3 by 4 grid.
Or, 3 x 4 =12

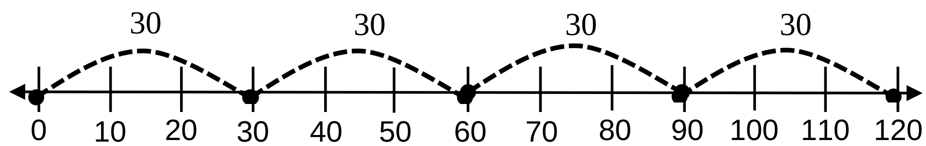
BASIC MULTIPLICATION MATH FACT MODELS

Large Whole Number Multiplication and Division Diagrams

The basic math fact models shown above are exactly the same physical models for larger digit numbers – _regardless of the size of the whole numbers_. However, many elementary and middle school students do not transition these same visual models from their math fact work to computing products of two and three digit numbers. Of course, students are required to learn algorithmic methodology as per Common Core State Standards (CCSS), Texas Essential Knowledge and Skill standards (TEKS) or any other state standard. However, when students compute the products of larger numbers, they are often so focused on the computational algorithm that they do not ingrain or realize that their solution is the same physical model previously learned for basic multiplication math fact models in third grade.

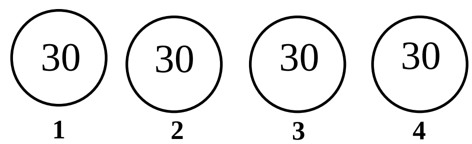
Group Models:

A.



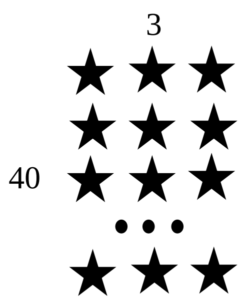
4 equal groups of 30 equals 120. Or, 4 x 30 =120

B.

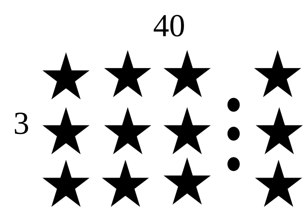


4 equal groups with 30 in each group equals 120. Or, 4 x 30 =120

Array Model:

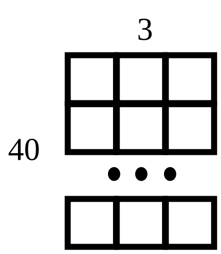


40 equal rows of 3.
Or, 40 x 3 =120

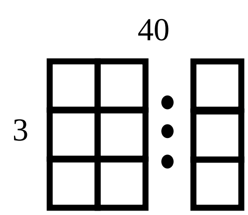


3 equal rows of 40.
Or, 3 x 40 =120

Area Model:



40 by 3 grid.
Or, 40 x 3 =120



3 by 40 grid.
Or, 3 x 40 =120

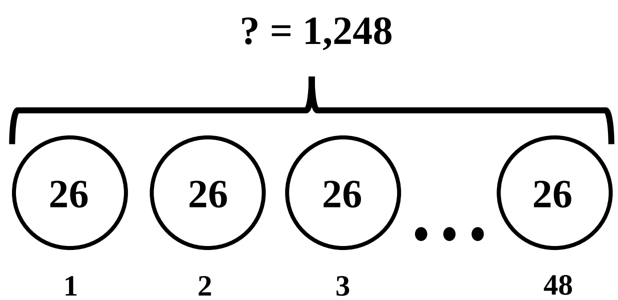
• • • Designates more equal groups or objects in between these groups.

MULTIPLICATION MODELS – Larger Numbers

Large Whole Number Multiplication and Division Diagrams

Once the basic one-digit math facts are memorized, the multiplication and division algorithms are relatively easily learned. Students become adept at multiplying larger whole numbers (i.e. 26 x 48 = ?). Again, despite algorithm proficiency, many students lose sight of the physical model (group, array and area) that they readily understood when learning their basic math facts. This situation is more troublesome for classroom teachers as students become 5th graders and middle school students. The whole numbers engaged in those grades are much larger than previously. Hence, students require an easily created model or tool to ingrain the physical meaning of large whole number multiplication.

For example, compute and show the meaning of 26 x 48 = ?.



$$\begin{array}{r} 26 \\ \times\ 48 \\ \hline 1{,}248 \end{array}$$

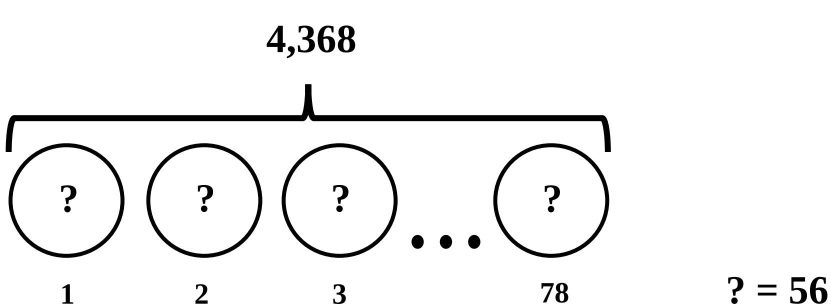
48 equal groups with 26 in each group equals 1,248. Or, 48 x 26 =1,248

NOTE: Alternatively, 26 equal groups with 48 in each group equals 1, 248.

Students should practice 2 x 1, 3 x1, 2 x 2, 3 x 2, etc. whole number multiplication creating the diagram above on every third or fourth problem, so the physical meaning of the computation algorithm is present in their learning. Since it is a multiplication computation, the **product** (i.e. 1,248 above) will always be the final number written on the diagram.

Of course, the same diagram may be used for division. However, the diagram is completed differently, since the **dividend** (i.e. the product above) is a known quantity in division. When using the model for division, a student is solving for the quotient or divisor since either number is interchangeable in the division process (e.g. 1,248 ÷ 48 = 26 or 1,248 ÷ 26 = 48).

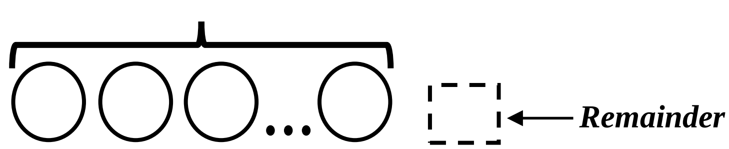
***For example, compute and show the meaning of* 4,368 ÷ 78 = ?**



$$\begin{array}{r} 56 \\ 78\overline{)4{,}368} \end{array}$$

4,368 separated into 78 equal groups yields ? = 56 'objects' in each group.

If there is a remainder in the division solution, then the diagram can include a small rectangle to the left of the standard diagram to include the remainder.



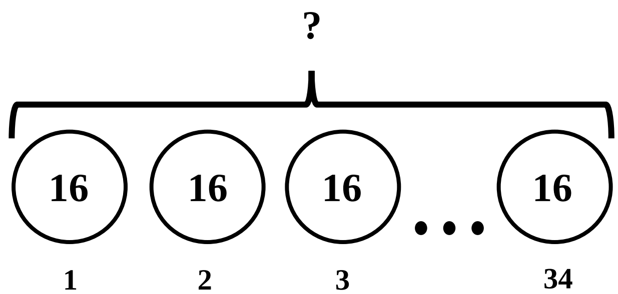
Large Whole Number Multiplication and Division Diagrams

Students can always understand the conceptual model of multiplication or division of any size whole number computation by drawing this simple diagram and filling in numbers in the appropriate places. ***But, this diagram possesses deceptive viability when students are solving word or story problems***. For instance, students can read the problem, draw the diagram, place numbers in the appropriate places and correctly solve the story problem since it is **visually** obvious that either the ***product* or the *quotient* is missing**.

Example 1: ***Solve the following story problem and create the correct operation diagram.***

Jim went to the movies for 34 consecutive Saturday evenings. He always spent exactly 16 dollars at the movie on popcorn, a soda and a ticket. How much money did Jim spend on his Saturday night movie-goings?

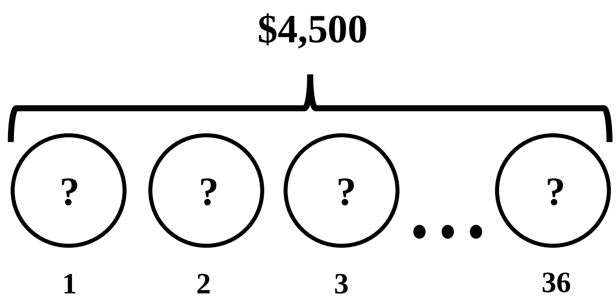
Solution: When the diagram is completed, the student discovers that a ***product*** is missing, and a verbose story problem is simplified to an easily drawn diagram representing a physical ***multiplication*** model – despite large whole numbers. Only the computation remains to solve the story problem.



? is a missing **product**, so the story problem ***must*** be a **multiplication** problem.

Example 2: ***Solve the following story problem and create the correct operation diagram.***

Luz and her husband purchased a new car for forty five hundred dollars. They made equal monthly payments for 36 months. What is the monthly payment for their new car?



? is a missing **quotient**, so the story problem ***must*** be a **division** problem.

Solution: As before, when the diagram is completed, the student discovers that the quotient is missing, and a developmental and complicated story problem is simplified to an easily drawn diagram representing the physical ***division*** model – despite large numbers. Only the computation remains to solve the story problem. If a remainder is involved, as explained earlier, add a small rectangle to the right to include its place in the diagram. It is important to note that this process solves difficult conceptual problems ***visually*** for the student, and the completed diagram may be readily used in logical reasoning to determine the correct solution.

For example: *At Forest North Elementary, 589 students will ride the bus to SeaWorld. If a maximum of 72 students are able to ride in one bus, how many buses are needed to travel to SeaWorld?*

Note: The diagram will show the student that remainder requires that an additional bus is needed.

For example: *Jerry bought a truck for $10,125. If he makes equal payments each month for 45 months, how much money does Jerry pay in the 20th month?*

Note: The diagram visually communicates that the same dollar amount is paid each and every month, not only in the 20th month. This type of division problem conceptually confuses many students.

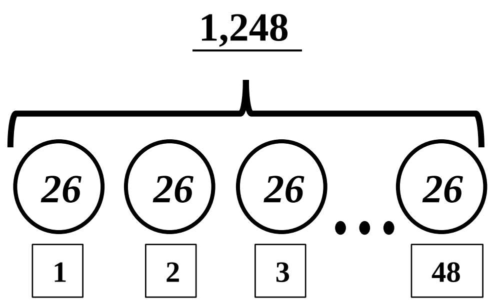
Student and/or Teacher Guided Practice Problems for Multiplication, Division and Story Problems

Whole Number Multiplication Diagram Practice Sheets

Directions: Compute the products. **Complete** the diagrams correctly. **Fill** in the blanks.

1.

$$\begin{array}{r} 26 \\ \times\ 48 \\ \hline 1{,}248 \end{array}$$

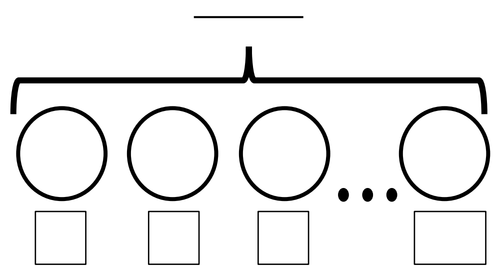


There are *48* equal groups of *26* totaling *1,248*

Note: *48 and 26 interchangeable on solution.*

2.

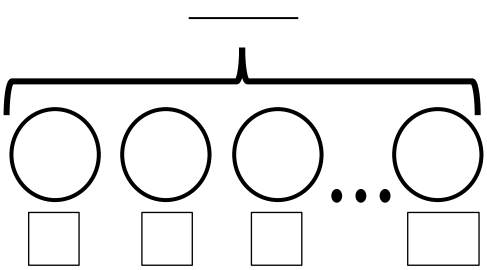
$$\begin{array}{r} 75 \\ \times\ 37 \\ \hline \end{array}$$



There are ____ equal groups of ____ totaling _____

3.

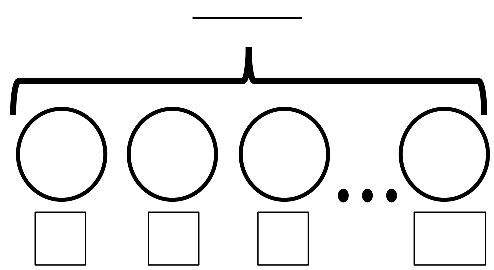
$$\begin{array}{r} 92 \\ \times\ 67 \\ \hline \end{array}$$



There are ____ equal groups of ____ totaling _____

4.

$$\begin{array}{r} 59 \\ \times\ 16 \\ \hline \end{array}$$

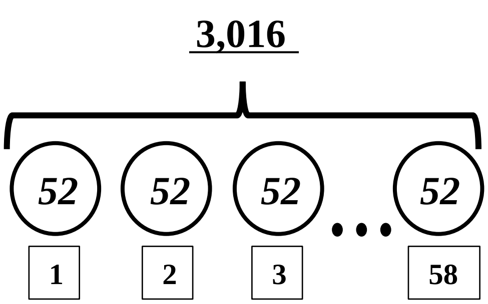


There are ____ equal groups of ____ totaling _____

Directions: Compute the products. **Draw** the diagrams correctly. **Fill** in the blanks.

1.

$$\begin{array}{r} 58 \\ \times\ 52 \\ \hline 3{,}016 \end{array}$$



There are *58* equal groups of *52* totaling *3,016*

Note: *58 and 52 interchangeable on solution. (Typical)*

2.

$$\begin{array}{r} 19 \\ \times\ 8 \\ \hline \end{array}$$

There are ____ equal groups of ____ totaling _____

3.

$$\begin{array}{r} 36 \\ \times\ 69 \\ \hline \end{array}$$

There are ____ equal groups of ____ totaling _____

4.

$$\begin{array}{r} 19 \\ \times\ 78 \\ \hline \end{array}$$

There are ____ equal groups of ____ totaling _____

Whole Number Multiplication Diagram Practice Sheets

Solutions

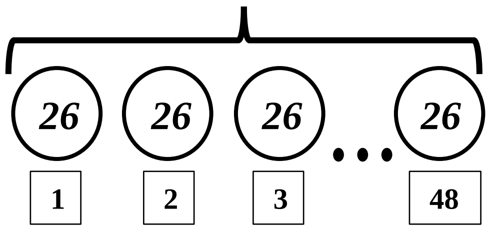
Directions: Compute the products. **Complete** the diagrams correctly. **Fill** in the blanks.

Solutions

1.

$$\begin{array}{r} 26 \\ \times\ 48 \\ \hline 1{,}248 \end{array}$$

1,248



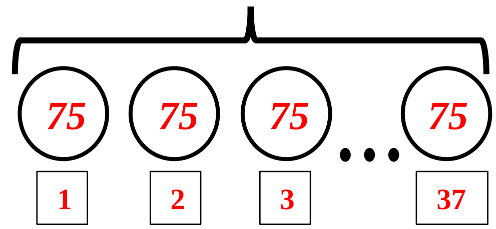
There are _48_ equal groups of _26_ totaling _1,248_

Note: *48 and 26 interchangeable on solution. (Typical)*

2.

$$\begin{array}{r} 75 \\ \times\ 37 \\ \hline 2{,}775 \end{array}$$

2,775



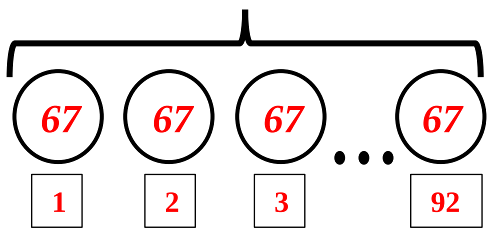
There are _37_ equal groups of _75_ totaling _2,775_

Note: *37 and 75 interchangeable on solution. (Typical)*

3.

$$\begin{array}{r} 92 \\ \times\ 67 \\ \hline 6{,}164 \end{array}$$

6,164

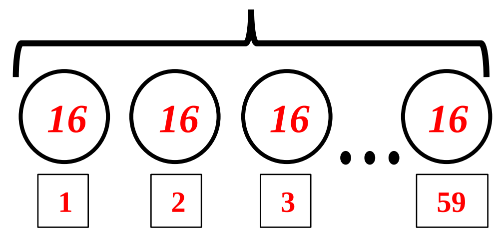


There are _92_ equal groups of _67_ totaling _6,164_

4.

$$\begin{array}{r} 59 \\ \times\ 16 \\ \hline 944 \end{array}$$

944



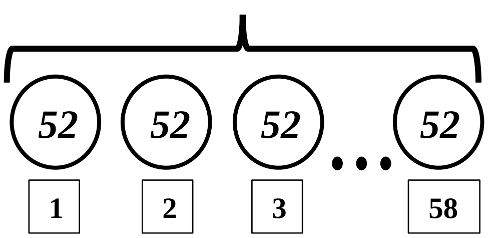
There are _59_ equal groups of _16_ totaling _944_

Directions: Compute the products. **Draw** the diagrams correctly. **Fill** in the blanks.

1.

$$\begin{array}{r} 58 \\ \times\ 52 \\ \hline 3{,}016 \end{array}$$

3,016



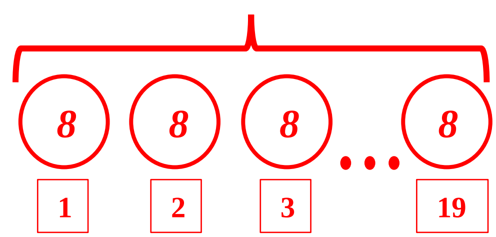
There are _58_ equal groups of _52_ totaling _3,016_

Note: *58 and 52 interchangeable on solution. (Typical)*

2.

$$\begin{array}{r} 19 \\ \times\ 8 \\ \hline 152 \end{array}$$

152



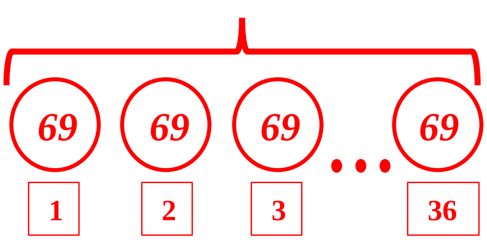
There are _19_ equal groups of _8_ totaling _152_

Note: *19 and 8 interchangeable on solution. (Typical)*

3.

$$\begin{array}{r} 36 \\ \times\ 69 \\ \hline 2{,}484 \end{array}$$

2,484

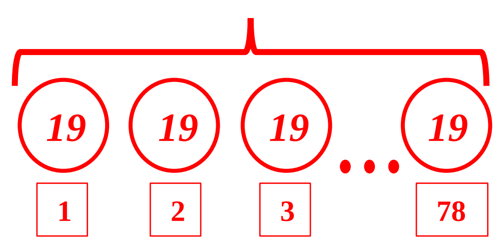


There are _36_ equal groups of _69_ totaling _2,484_

4.

$$\begin{array}{r} 19 \\ \times\ 78 \\ \hline 1{,}482 \end{array}$$

1,482



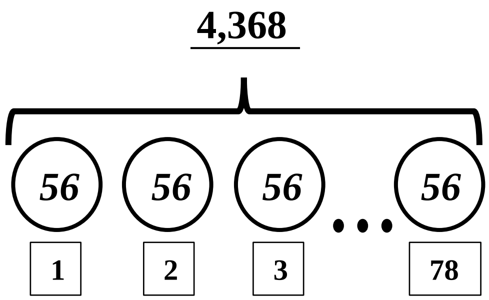
There are _78_ equal groups of _19_ totaling _1,482_

Whole Number Division Diagram Practice Sheets

Directions: Compute the quotients. **Complete** the diagrams correctly. **Fill** in the blanks.

1.

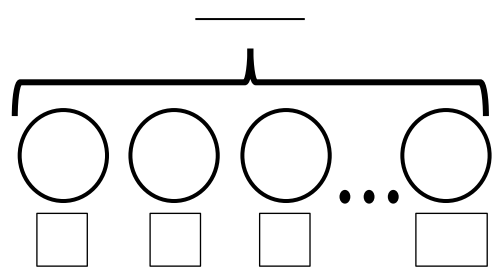
$$\begin{array}{r} 56 \\ 78\overline{)4{,}368} \end{array}$$



4,368 separated in *78* equal groups *56* in each group.

2.

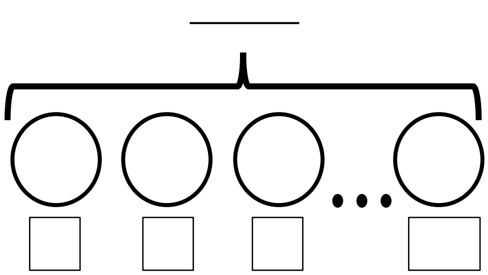
$$32\overline{)1{,}440}$$



______ separated in _____ equal groups _____ in each group.

3.

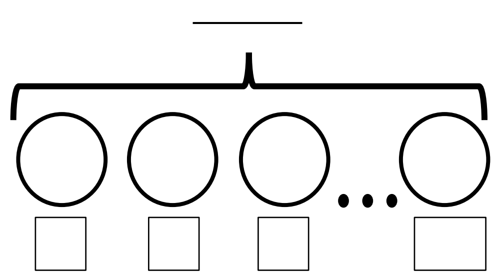
$$9\overline{)207}$$



______ separated in _____ equal groups _____ in each group.

4.

$$51\overline{)1{,}785}$$

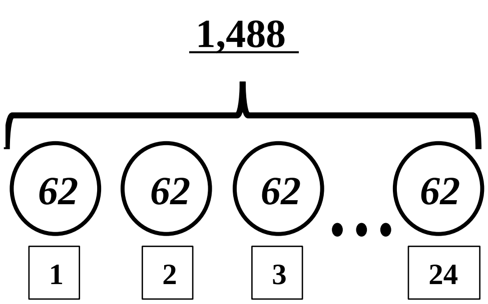


______ separated in _____ equal groups _____ in each group.

Directions: Compute the quotients. **Draw** the diagrams correctly. **Fill** in the blanks.

1.

$$\begin{array}{r} 62 \\ 24\overline{)1{,}488} \end{array}$$



1,448 separated in *24* equal groups *62* in each group.

2.

$$43\overline{)1{,}204}$$

______ separated in _____ equal groups _____ in each group.

3.

$$14\overline{)322}$$

______ separated in _____ equal groups _____ in each group.

4.

$$9\overline{)504}$$

______ separated in _____ equal groups _____ in each group.

Whole Number Division Diagram Practice Sheets

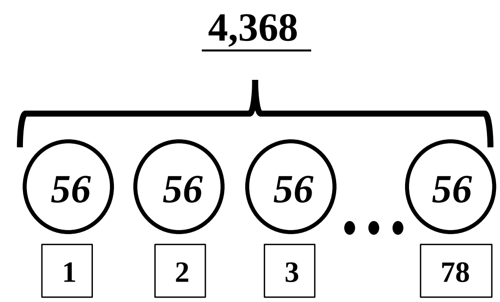
Solutions

Directions: **Compute** the quotients. **Complete** the diagrams correctly. **Fill** in the blanks.

Solutions

1.

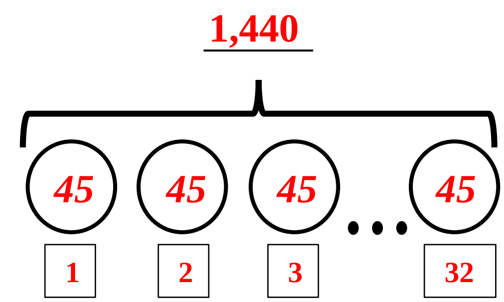
$$78\overline{)4{,}368} = 56$$



4,368 separated in 78 equal groups 56 in each group.

2.

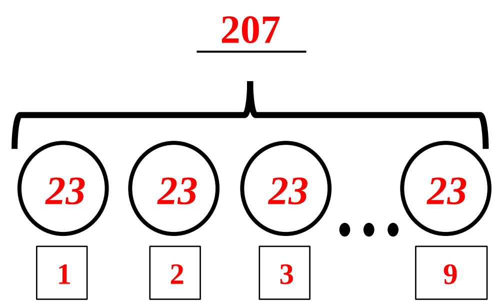
$$32\overline{)1{,}440} = 45$$



1,440 separated in 32 equal groups 45 in each group.

3.

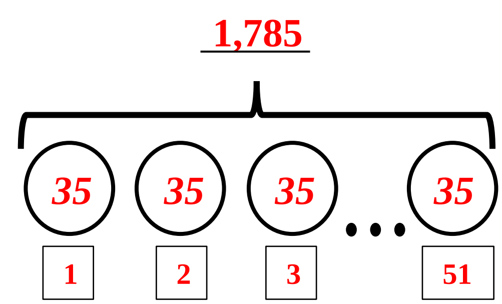
$$9\overline{)207} = 23$$



207 separated in 9 equal groups 23 in each group.

4.

$$51\overline{)1{,}785} = 35$$

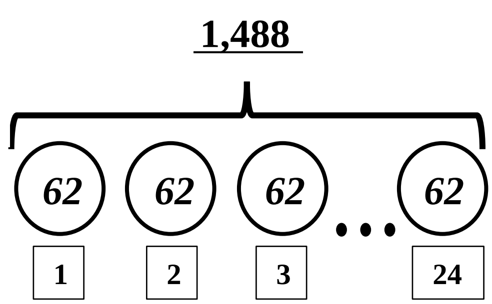


1,785 separated in 51 equal groups 35 in each group.

Directions: **Compute** the quotients. **Draw** the diagrams correctly. **Fill** in the blanks.

1.

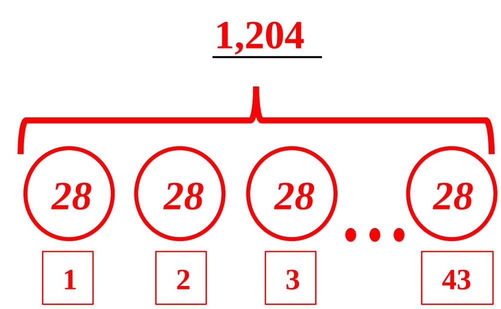
$$24\overline{)1{,}488} = 62$$



1,448 separated in 24 equal groups 62 in each group.

2.

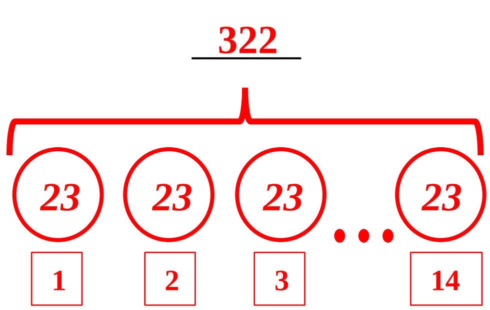
$$43\overline{)1{,}204} = 28$$



1,204 separated in 43 equal groups 28 in each group.

3.

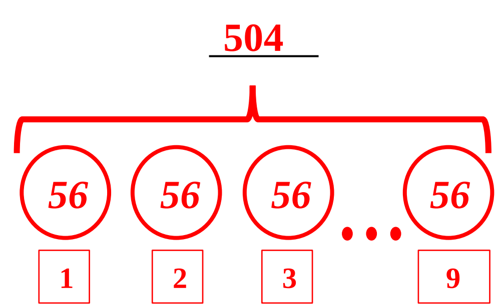
$$14\overline{)322} = 23$$



322 separated in 14 equal groups 23 in each group.

4.

$$9\overline{)504} = 56$$



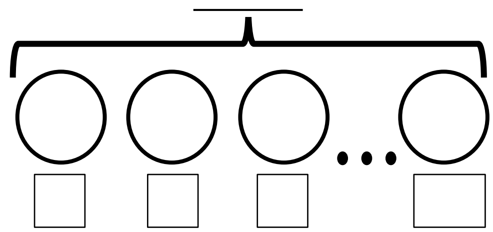
504 separated in 9 equal groups 56 in each group.

Whole Number Multiplication & Division Diagram Story Problems

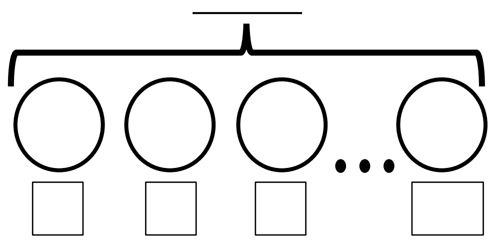
Directions: Solve the problem. Complete or Create the diagram. Problem 1 is done for you.

1. Joseph placed an equal number of bricks in 43 stacks. If there are 26 bricks in each stack, how many bricks did Joseph stack?

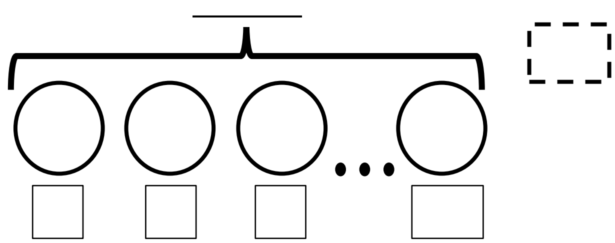
$$\begin{array}{r} 26 \\ \times\ 43 \\ \hline 1{,}118 \end{array}$$



2. Jesus earns 35 dollars each month from completing his weekly chores. He puts the money in his bank account at First National Savings and Loan. What is that total amount of money Jesus will save if he saves the same amount each month for 48 months?



3. Betty is shopping for a newer car to drive. She decided to purchase a used car for 5,700 dollars. If she pays an equal amount for 60 months, what is her car payment each month?



4. Forest East Middle School is sponsoring a nightly reading contest. Ana wants to participate. Her book has 540 pages. If she reads 45 pages each night, how many nights will it take her to complete her book?

5. Yasmin ran 75 kilometers a week training for a marathon race. She trained for 22 weeks. How many kilometers did Yazmin run during her marathon training?

Whole Number Multiplication & Division Diagram Story Problems

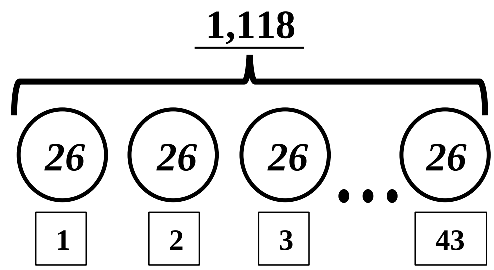
Solutions

Directions: Solve the problem. Complete or Create the diagram. Problem 1 is done for you.

Solutions

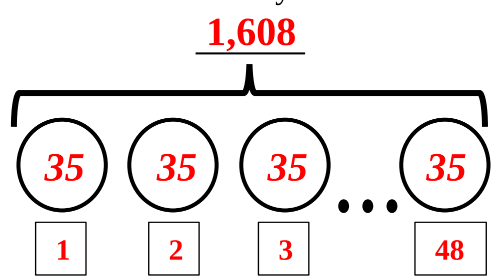
1. Joseph placed an equal number of bricks in 43 stacks. If there are 26 bricks in each stack, how many bricks did Joseph stack?

$$\begin{array}{r} 26 \\ \times\ 43 \\ \hline 1{,}118 \end{array}$$



2. Jesus earns 35 dollars each month from completing his weekly chores. He puts the money in his bank account at First National Savings and Loan. What is that total amount of money Jesus will save if he saves the same amount each month for 48 months?

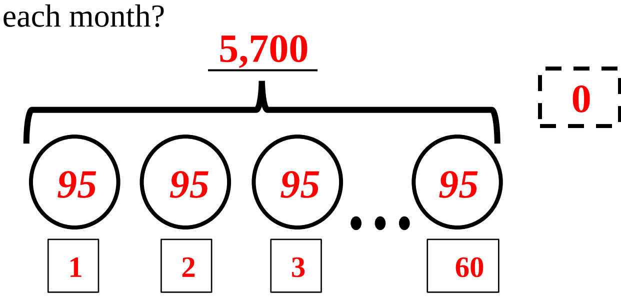
$$\begin{array}{r} 35 \\ \times\ 48 \\ \hline 1{,}680 \end{array}$$



3. Betty is shopping for a newer car to drive. She decided to purchase a used car for 5,700 dollars. If she pays an equal amount for 60 months, what is her car payment each month?

$$60\overline{)5{,}700}\quad 95$$

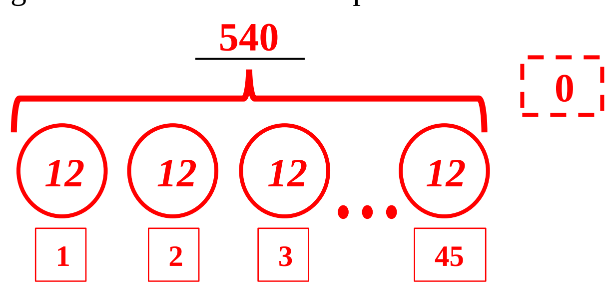
Remainder is 0



4. Forest East Middle School is sponsoring a nightly reading contest. Ana wants to participate. Her book has 540 pages. If she reads 45 pages each night, how many nights will it take her to complete her book?

$$45\overline{)540}\quad 12$$

Remainder is 0



5. Yasmin ran 75 kilometers a week training for a marathon race. She trained for 22 weeks. How many kilometers did Yazmin run during her marathon training?

$$\begin{array}{r} 22 \\ \times\ 75 \\ \hline 1{,}650 \end{array}$$