Multiplication Factor Strings using the Compression Method

Writing all factors in an organized manner for numbers between 1 and 60.

4th – 8th Grade

Authored by:

**Amara 4 Education
Curriculum Writers**

Executive Summary

This short white paper describes the Compression Method for finding factor strings using simple divisibility rules and spaced repetition. State standard strands across the United States stipulate that students should possess immediate recall of their four operation facts for digits 1 through 12. In general, it is imperative to note that students must possess rapid recall of all four-math fact operations. This task is not as demanding as an entry-level educator may believe. Using a structured and consistent approach described in white papers located at the web address in the footer, students will master their basic facts rapidly.

However, organizing the multiplication factors for any given number so factors are readily listed for specific applications is also important. A methodology to accomplish this task is the "Compression Method." The pedagogical process is aptly called the Compression Method because factors in a specific multiplication string are computed the outside – inward. The methodology is extremely simple so that any teacher of any experience level can provide instruction using spaced repetition over the course of two to three weeks. A further recommendation is that teachers combine prime and composite number recognition in the process, so another important skill is mastered in arithmetic student learning.

The divisibility rules needed should be learned prior to using the Compression Method, but the divisibility rules for 4, 7, and 8 are complicated for students to remember. Math facts that possess 4, 7 and 8 only appear in isolation in the following math facts: 4 x 7, 4 x 8, 7 x 8, 4 x 4, 7 x 7 or 8 x 8. Hence, it is recommended that the divisibility rules for 2, 3, 5, 6, 9 and 10 are the basic arithmetic rules needed to master most of the numbers between 1 and 60. In order to be both effective and efficient, students should have a basic, working practice of their multiplication facts.

The compression facts provide students the rapid means to write the math facts for any number in an organized, linear and useable manner. For example, the Compression Method produces all the factors for the number 36 as shown below:

$$36: \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$

This instructional pedagogical process can begin in 3rd grade, but it is highly recommended that this process be a non-negotiable in fourth grade classrooms during the spring semester. Mastery of this numeracy skill will be advantageous for both 5th grade and middle school students when they are reducing fractions to lowest terms or simplest form. However, the other benefits of this methodology extends far beyond equivalent fractions. The Compression Method methodology provides students with accelerated numeracy understanding of multiplication factors and connects multiple math facts to a single number as shown in the above example using 36.

Multiplication Factor Strings using the Compression Method

Memorization of basic math facts of all four operations is essential arithmetic knowledge. Unlike subtraction and division, multiplication and addition fact mastery is usually easier for most students because of the reduced number of facts to memorize. The commutative property of addition and multiplication cut the number of memorized facts in half since 5 + 6 yields the same sum as 6 + 5, and 6 x 5 and 5 x 6 produce the same product in multiplication. Daily numeracy programs like Formative Loop include a five-minute daily writing assessment. The program's writing element is critical for many students to memorize and ingrain their math fact operations. The digital input and analysis side of the Formative Loop numeracy program affords classroom teachers with a tool to monitor and heighten student performance in real time.

However, as students begin fourth grade, it is also important to write factors for a given number in a linear and organized manner. When students write the factors for a number, they often write them in pairs. For example, given the number 24, many students write the math facts in the following manner: (1, 24); (4, 6); (3, 8) and (2, 12). Many times, elementary students miss a fact pair or two when instructed with this methodology. Additionally, factors written in pairs do not always provide pragmatic use. The Compression Method yields an organized factor string that not only lists the number's factors, but factors are written in an ascending order. Hence, given the number 24, the factor string is written in the following linear manner: 24: {1, 2, 3, 4, 6, 8, 12, 24}. This process connects all the factors for 24 in an organized and usable manner. It also helps students ingrain their multiplication math facts in long-term memory.

This short pedagogy paper provides the means for a teacher of any experience to instruct students using Compression Method methodology to write a string of factors for any number between 1 and 60. Prior to using this method, students must be proficient using the disability rules for 2, 3, 5, 6, 9, and 10. The divisibility rules for 4, 7 and 8 are more difficult for students to remember; however, those three numbers only appear together in the multiplication facts of 4 x 7; 4 x 8; 7 x 8; 4 x 4; 7 x 7 or 8 x 8. Otherwise, those 3 numbers appear as a factor pair with 2, 3, 5, 6, 9, or 10. With small amounts of practice, students easily master the most common divisibility rules for 2, 3, 5, 6, 9 and 10. These rules are summarized below:

Divisibility Rules for 2, 3, 5, 6, 9, 10		
Number	**Rule**	**Example**
2	If the one's digit is even, then the number is divisible by 2.	Given the number 38. Even numbers: (0, 2, 4, 6, 8) The '8' is an even number. Hence, 38 is divisible by 2.
3	If the digits of a number sum to a multiple of 3, then the number is divisible by 3.	Given the number, **126.** **1** + **2** + **6** = 9. 9 divides by 3 without a remainder. Hence, 126 is divisible by 3.
5	If the one's digit is either a 5 or 0, then the number is divisible by 5.	Given the numbers **45** and **70**. 45 ends in '5' and 70 ends in '0'. Hence, 45 and 70 are divisible by 5
6	If a number is divisible by **both** 2 and 3, then the number is divisible by 6.	Given the number **18**. 18 is divisible by 2 and 3. Hence, it is divisible by 6, too
9	If the digits of a number sum to a multiple of 9, then the number is divisible by 9.	Given the number, **234.** **2** + **3** + **4** = 9. 9 divides by 9 without a remainder. Hence, 234 is divisible by 9.
10	If the one's digit is either a 0, then the number is divisible by 10.	Given the number **190**. 190 ends in '0'. Hence, 190 is divisible by 10.

 www.amara4education.com

Multiplication Factor Strings using the Compression Method

Compression Method Illustrated

The '*Compression Method*' is duly named because factors are computed from the outside - inward (→ ←). Hence, students utilize basic divisibility rules and math fact knowledge to write ALL the factors for a given number (from 1 to 60) from the extremities of the brackets to the factor string's center.

Example 1: Find all the factors for the number 12:

      Step 1: Write the number 12 with brackets.   **12: {   }**

      Step 2: Write the *identity factor pair* for the number 12:   **12: {1,   , 12}**

            Note: The only possible factors left that are between 2 and 11, inclusive.

      Step 3: Is the number 12 even? Yes, divisible by 2.   **12: {1, 2,   6, 12}**

            Note: The only possible factors left that are between 3 and 5, inclusive.

      Step 4: Is the number 12 divisible by 3? Yes. Check, 1 + 2 is 3.   **12: {1, 2, 3, 4, 6, 12}**

            Note: There are no whole numbers between 3 and 4. Done.

.

Example 2: Find all the factors for the number 45:

      Step 1: Write the number 45 with brackets.   **45: {   }**

      Step 2: Write the identity factor pair for the number 45:   **45: {1,   , 45}**

            Note: The only factors left that are possible are between 2 and 44, inclusive.

      Step 3: Is the number 45 even? No! Then, not divisible by 2.   **45: {1,   , 45}**

      Step 4: Is the number 45 divisible by 3? Check, 4 + 5 is 9.   **45: {1, 3,   15, 45 }**

            Note: The only factors left that are possible are between 4 and 14, inclusive

      Step 5: Is the number 45 divisible by 4? No, math fact knowledge.   **45: {1, 3,   15, 45 }**

      Step 6: Is the number 45 divisible by 5? Yes, divisible by 5.   **45: {1, 3, 5, 9, 15, 45 }**

            Note: The only factors that are possible are between 6 and 8, inclusive.

Continuing, is the number divisible by 6, 7 or 8? No! It is not. The factor string for 45 is complete.

Multiplication Factor Strings using the Compression Method

Spaced Repetition – A Synopsis

One of the most effective and efficient pedagogical methods for a classroom teacher to instruct students is via spaced repetition. This pedagogy ensures that _**ALL**_ students view a skill a threshold number of repetitions to ingrain mastery. Spaced repetition takes place during the math lesson – usually at the very beginning. It is a short 5 to 10 minute review of previously taught skills (usually 3 to 5 skills) presented in a rapid and lively manner prior to the onset of the daily core lesson. During a typical spaced repetition session, the teacher is actively monitoring students – especially students that have **NOT** mastered the skill. As the teacher observes the remaining few students master a particular math skill, that skill is replaced with another skill that has been previously taught. Hence, the process is not static, it is a sequential and dynamic process throughout the school year.

Spaced repetition may be used for standalone math skills like place value, even/odd, rounding, etc. However, the daily pedagogy may also be used for the skill mastery of more intricate process skills such as mastering the computation of lowest common denominators (LCD), pre-prepared measurement exercises, and factor string mastery using the compression method. It is a little more involved in these latter skill sets, but it is highly effective with adequate teacher understanding, preparation and professional practice.

Using Spaced Repetition for Compression Method Factor Strings

If the teacher presents the process of computing factor pairs in small daily chunks starting with the numbers 1 through 5, students become very adept at writing factor strings. Then, the teacher should require students to **cumulatively** write factor strings for the next five numbers (6 through 10), and then the next five (11 through 15). For time efficiency, once students **master** the factor strings for numbers 1 through 5, the teacher should continue to the next five numbers (i.e. 6-10). Eliminate writing the factors for numbers 1 through 5. Continue introducing two groups of 5 numbers in stages as mastery of the lower group is achieved until the factor string for 55 through 60 is reached. Students can use notebook paper or a daily measurement warm-up that may easily be flipped over to the blank side for the spaced repetition session. This instructional daily process will take consistent practice for 2 to 3 weeks to complete for all factor strings to 60. See the illustration below for numbers 1 – 15. Note: Factor String # 4 (Numbers 11 through 20), Factor String # 5 (Numbers 16 – 25), Factor String # 6 (Numbers 21 – 30), and so on.

Determining whether the number is either 'prime - **P**' or 'composite - **C**' is an easy add-on skill. Use a kinesthetic tactile method to identify prime numbers such as the two-finger 'peace' sign or the University of Texas Longhorns "Hook-em Horns" sign. Any kinesthetic method will work as long as students recognize numbers with only two factors are classified as prime numbers. A skill practice sheet is attached as a final assessment or intermediate homework for student practice computing all factors for the numbers 1 through 60, inclusive.

Sequencing of Factor String for Spaced Repetition – *Numbers 1 – 15 shown*

Factor Strings # 1		
1: { } ********		
2: { } **P C**		
3: { } **P C**		
4: { } **P C**		
5: { } **P C**		

Factor Strings # 2				
1: { } **P C**	**6:** { } **P C**			
2: { } **P C**	**7:** { } **P C**			
3: { } **P C**	**8:** { } **P C**			
4: { } **P C**	**9:** { } **P C**			
5: { } **P C**	**10:** { } **P C**			

Factor Strings # 3				
6: { } **P C**	**11:** { } **P C**			
7: { } **P C**	**12:** { } **P C**			
8: { } **P C**	**13:** { } **P C**			
9: { } **P C**	**14:** { } **P C**			
10: { } **P C**	**15:** { } **P C**			

**** NOTE: The number one (1) is **not** classified as either prime or composite. It is a 'special' number, in this situation.

Computing Factor Strings and Prime - Composite Identification

Write all the factors for each number using the 'compression method' – see example.   After all the factors have been listed for each number, circle the correct number label: "Prime or Composite." Review example.

Example: **12** .. {1,   12};   then {1, 2,   6, 12};   then, **{1, 2, 3, 4, 6, 12}**   Prime or (Composite)

1: _____   ***NOT*** ***Prime or Composite***

2: _____   **Prime** or **Composite**

3: _____   **Prime** or **Composite**

4: _____   **Prime** or **Composite**

5: _____   **Prime** or **Composite**

6: _____   **Prime** or **Composite**

7: _____   **Prime** or **Composite**

8: _____   **Prime** or **Composite**

9: _____   **Prime** or **Composite**

10: _____   **Prime** or **Composite**

11: _____   **Prime** or **Composite**

12: _____   **Prime** or **Composite**

13: _____   **Prime** or **Composite**

14: _____   **Prime** or **Composite**

15: _____   **Prime** or **Composite**

16: _____   **Prime** or **Composite**

17: _____   **Prime** or **Composite**

18: _____   **Prime** or **Composite**

19: _____   **Prime** or **Composite**

20: _____   **Prime** or **Composite**

21: _____   **Prime** or **Composite**

22: _____   **Prime** or **Composite**

23: _____   **Prime** or **Composite**

24: _____   **Prime** or **Composite**

25: _____   **Prime** or **Composite**

26: _____   **Prime** or **Composite**

27: _____   **Prime** or **Composite**

Computing Factor Strings and Prime - Composite Identification

28: _____   **Prime** or **Composite**

29: _____   **Prime** or **Composite**

30: _____   **Prime** or **Composite**

31: _____   **Prime** or **Composite**

32: _____   **Prime** or **Composite**

33: _____   **Prime** or **Composite**

34: _____   **Prime** or **Composite**

35: _____   **Prime** or **Composite**

36: _____   **Prime** or **Composite**

37: _____   **Prime** or **Composite**

38: _____   **Prime** or **Composite**

39: _____   **Prime** or **Composite**

40: _____   **Prime** or **Composite**

41: _____   **Prime** or **Composite**

42: _____   **Prime** or **Composite**

43: _____   **Prime** or **Composite**

44: _____   **Prime** or **Composite**

45: _____   **Prime** or **Composite**

46: _____   **Prime** or **Composite**

47: _____   **Prime** or **Composite**

48: _____   **Prime** or **Composite**

49: _____   **Prime** or **Composite**

50: _____   **Prime** or **Composite**

51: _____   **Prime** or **Composite**

52: _____   **Prime** or **Composite**

53: _____   **Prime** or **Composite**

54: _____   **Prime** or **Composite**

55: _____   **Prime** or **Composite**

56: _____   **Prime** or **Composite**

60: _____   **Prime** or **Composite**

Computing Factor Strings and Prime - Composite Identification

Write all the factors for each number using the 'compression method' – see example.   After all the factors have been listed for each number, circle the correct number label: "Prime or Composite." Review example.

Example: 12 .. {1,   12};   then {1, 2,   6, 12};   then, **{1, 2, 3, 4, 6, 12}**   Prime or (**Composite**)

1: _____ **(1)** _____   *NOT Prime or Composite*

2: _____ **(1, 2)   only even number that is prime.** _____   (Prime) or Composite

3: _____ **(1, 3)** _____   (Prime) or Composite

4: _____ **(1, 2, 4)** _____   Prime or (Composite)

5: _____ **(1, 5)** _____   (Prime) or Composite

6: _____ **(1, 2, 3, 6)** _____   Prime or (Composite)

7: _____ **(1, 7)** _____   (Prime) or Composite

8: _____ **(1, 2, 4, 8)** _____   Prime or (Composite)

9: _____ **(1, 3, 9)** _____   Prime or (Composite)

10: _____ **(1, 2, 5, 10)** _____   Prime or (Composite)

11: _____ **(1, 7)** _____   (Prime) or Composite

12: _____ **(1, 2, 3, 4, 6, 12)** _____   Prime or (Composite)

13: _____ **(1, 13)** _____   (Prime) or Composite

14: _____ **(1, 2, 7, 14)** _____   Prime or (Composite)

15: _____ **(1, 3, 5, 15)** _____   Prime or (Composite)

16: _____ **(1, 2, 4, 8, 16)** _____   Prime or (Composite)

17: _____ **(1, 17)** _____   (Prime) or Composite

18: _____ **(1, 2, 3, 6, 9, 18)** _____   Prime or (Composite)

19: _____ **(1, 19)** _____   (Prime) or Composite

20: _____ **(1, 2, 4, 5, 10, 20)** _____   Prime or (Composite)

21: _____ **(1, 3, 7, 21)** _____   Prime or (Composite)

22: _____ **(1, 2, 11, 22)** _____   Prime or (Composite)

23: _____ **(1, 23)** _____   (Prime) or Composite

24: _____ **(1, 2, 3, 4, 6, 8, 12, 24)** _____   Prime or (Composite)

25: _____ **(1, 5, 25)** _____   Prime or (Composite)

26: _____ **(1, 2, 13, 26)** _____   Prime or (Composite)

27: _____ **(1, 3, 9, 27)** _____   Prime or (**Composite**)

Computing Factor Strings and Prime - Composite Identification

28:   **(1, 2, 4, 7, 14, 28)**   Prime or (Composite)

29:   **(1, 29)**   (Prime) or Composite

30:   **(1, 2, 3, 5, 6, 10, 15, 30)**   Prime or (Composite)

31:   **(1, 31)**   (Prime) or Composite

32:   **(1, 2, 4, 8, 16, 32)**   Prime or (Composite)

33:   **(1, 3, 11, 33)**   Prime or (Composite)

34:   **(1, 2, 17, 34)**   Prime or (Composite)

35:   **(1, 5, 7, 35)**   Prime or (Composite)

36:   **(1, 2, 3, 4, 6, 9, 12, 18, 36)**   Prime or (Composite)

37:   **(1, 37)**   (Prime) or Composite

38:   **(1, 2, 19, 38)**   Prime or (Composite)

39:   **(1, 3, 13, 39)**   Prime or (Composite)

40:   **(1, 2, 4, 5, 8, 10, 20, 40)**   Prime or (Composite)

41:   **(1, 37)**   (Prime) or Composite

42:   **(1, 2, 3, 14, 21, 42)**   Prime or (Composite)

43:   **(1, 43)**   (Prime) or Composite

44:   **(1, 2, 4, 11, 22, 44)**   Prime or (Composite)

45:   **(1, 3, 5, 9, 15, 45)**   Prime or (Composite)

46:   **(1, 2, 23, 46)**   Prime or (Composite)

47:   **(1, 47)**   (Prime) or Composite

48:   **(1, 2, 3, 4, 6, 8, 12, 16, 48)**   Prime or (Composite)

49:   **(1, 7, 49)**   Prime or (Composite)

50:   **(1, 2, 5, 10, 25, 50)**   Prime or (Composite)

51:   **(1, 3, 17, 51)**   Prime or (Composite)

52:   **(1, 2, 26, 52)**   Prime or (Composite)

53:   **(1, 53)**   (Prime) or Composite

54:   **(1, 2, 3, 6, 9, 18, 27, 54)**   Prime or (Composite)

55:   **(1, 5, 11, 55)**   Prime or (Composite)

56:   **(1, 2, 7, 8, 28, 56)**   Prime or (Composite)

60:   **(1, 2, 3, 4, 5, 6, 10, 12, 15, 30, 60)**   Prime or (Composite)